New Correlations for Friction Factor and Nusselt Number Prediction for Laminar Flow in a Circular Duct having only Spiral Corrugations & only Twisted Tapes with Oblique Teeth

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Abstract— The experimental friction factor and Nusselt number data for laminar flow through a circular duct having only spiral corrugationsand only twisted tapes with oblique teeth have been correlated by new predictive correlations. The major findings of the experimental investigation and subsequest development of correlations are that these inserts are useful for practical applications in heat exchangers used in chemical plants and the correlationspredict data satisfactorily well within the permissible industrial limits.

Keywords— : Laminar Flow, Forced Convection, Spiral Corrugations, Twisted Tapes, Oblique Teeth, Swirl Flow

I. Introduction

Passive heat transfer techniques like spiral corrugations and twisted tapes with oblique teeth, as shown in Figure 1 and Figure 2, are used to mix the gross flow effectively in laminar flow to reduce the thermal resistance in the core flow through the channel. Edwards and Sheriff [1] and Emerson [2] worked with heat transfer enhancement in pipe flow. Uttawar and Raja Rao[3] carried out experiments with seven different wire coil insert s.Variants of twisted-tape [5-7] have been used.

Laminar flow friction factor and Nusselt number correlations for only spiral corrugations and only twisted tapes in circular ducts have been developed from experimental data [8] and these are presented here.

II. Experimental Set-Up, Operating Procedure And Data Reduction

The heat transfer and pressure drop measurements were taken in nine test sections, three each with 13 mm, 16 mm and 19 mm ID brass tubes. All nine tubes had 1 mm thickness and 2m length. Figure 3 shows the self-explanatory experimental rig. The usual fabrication method of the test section and test set-up, the operating procedure and data reduction method have been described well in [8]. Servotherm medium oil of Indian Oil Corporation was used as the working fluid. Thermocouples were installed on the duct outside wall by brazing. Duct inside wall temperatures were evaluated by calculating duct-wall temperature drop from the one-dimensional radial heat conduction equation. Peripherally local temperatures in an axial station were arithmetically averaged to get axially local temperature and Nusselt number. Then axially local Nusselt numbers were averaged by trapezoidal rule. Fanning Friction factor evaluated. Experimental uncertainty was determined by the method of Kline and McClintock [9].

Further details of the experimental work and the experimental data may be obtained from [8].

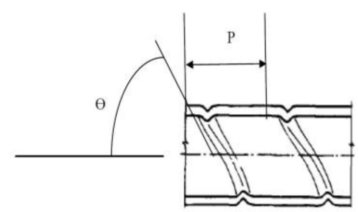

Fig. 1: Spiral Corrugation

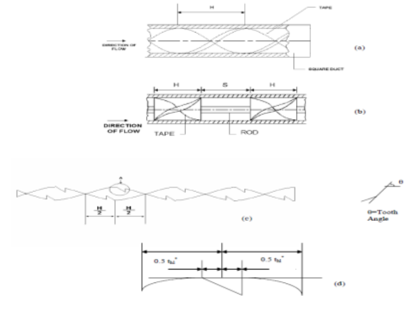

Fig. 2: Twisted Tape with and without Oblique Teeth

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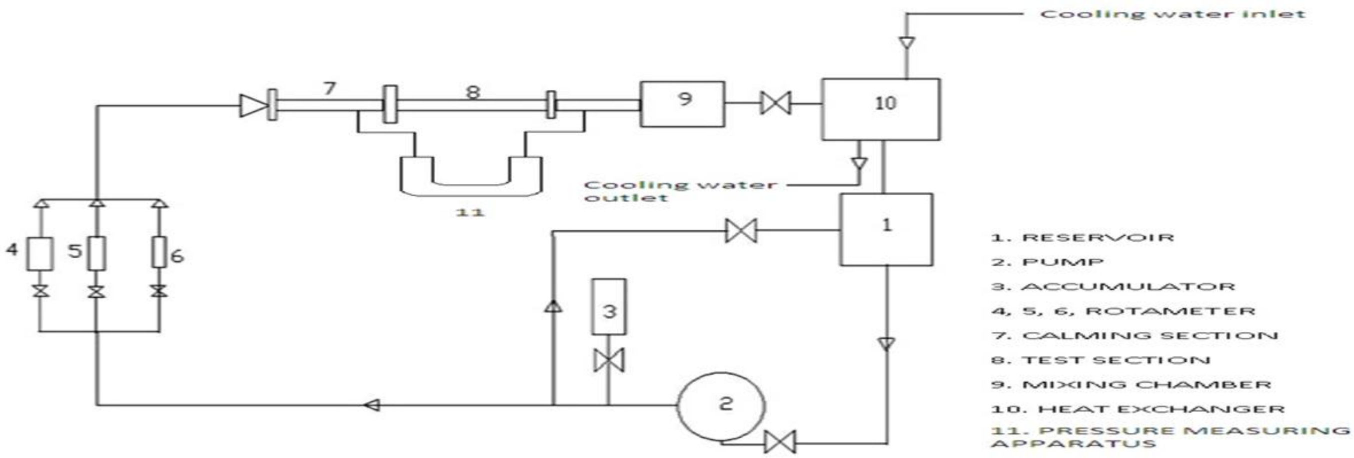

Fig. 3: Experimental Set-Up

III. Correlations

The correlations for Nusselt number and friction factor have been developed by log-linear regression analysis.

Correlation for Nusselt number for combined spiral corrugation & twisted-tape with oblique teeth in basic format is given by:

$$Nu = 5.172 \quad Gz^{c_1}\left(\frac{Re}{\sqrt{y}}\right)^{c_2} Pr^{c_3} \quad Gr^{c_4}\left(\frac{\mu_b}{\mu_w}\right)^{0.14} \quad t_{hl}{}^{c_5}(sin\theta)^{c_6}(sin\alpha)^{c_7} h_c{}^{c_8}$$

Where the constants are to be determined by log-linear regression analysis as follows:

$$\ln Nu = c_1 \ln Gz + c_2 \ln\left(\frac{Re}{\sqrt{y}}\right) + c_3 \ln Pr + c_4 \ln Gr + c_5 \ln t_{hl} + c_6 \ln sin\theta + c_7 \ln sin\alpha + c_8 \ln h_c$$

$$\sum_{i=1}^{N}\left(\begin{matrix} c_1 \ln Gz + c_2 \ln\left(\frac{Re}{\sqrt{y}}\right) + c_3 \ln Pr + c_4 \ln Gr + c_5 \ln t_{hl} + c_6 \ln sin\theta + c_7 \ln sin\alpha \\ + c_8 \ln h_c \quad - \ln Nu \end{matrix}\right)_i^2 = 0$$

$$\sum_{i=1}^{N}\left(c_1 \ln Gz + c_2 \ln\left(\frac{Re}{\sqrt{y}}\right) + c_3 \ln Pr + c_4 \ln Gr + c_5 \ln t_{hl} + c_6 \ln sin\theta + c_7 \ln sin\alpha + c_8 \ln h_c - \ln Nu\right) \ln Gz$$
$$= 0 \quad \sum_{i=1}^{N} c_1 \ln Gz \ln Gz + \sum_{i=1}^{N} c_2 \ln\left(\frac{Re}{\sqrt{y}}\right) \ln Gz + \sum_{i=1}^{N} c_3 \ln Pr \ln Gz + \sum_{i=1}^{N} c_4 \ln Gr \ln Gz + \sum_{i=1}^{N} c_5 \ln t_{hl} \ln Gz$$
$$+ \sum_{i=1}^{N} c_6 \ln sin\theta \ln Gz + \sum_{i=1}^{N} c_7 \ln sin\alpha \ln Gz + \sum_{i=1}^{N} c_8 \ln h_c \ln Gz = \sum_{i=1}^{N} \ln Nu \ln Gz$$

$$\sum_{i=1}^{N} c_1 \ln Gz \ln\left(\frac{Re}{\sqrt{y}}\right) + \sum_{i=1}^{N} c_2 \ln\left(\frac{Re}{\sqrt{y}}\right) \ln\left(\frac{Re}{\sqrt{y}}\right) + \sum_{i=1}^{N} c_3 \ln Pr \ln\left(\frac{Re}{\sqrt{y}}\right) + \sum_{i=1}^{N} c_4 \ln Gr \ln\left(\frac{Re}{\sqrt{y}}\right) + \sum_{i=1}^{N} c_5 \ln t_{hl} \ln\left(\frac{Re}{\sqrt{y}}\right) + \sum_{i=1}^{N} c_6 \ln sin\theta \ln\left(\frac{Re}{\sqrt{y}}\right) + \sum_{i=1}^{N} c_7 \ln sin\alpha \ln\left(\frac{Re}{\sqrt{y}}\right) + \sum_{i=1}^{N} c_8 \ln h_c \ln\left(\frac{Re}{\sqrt{y}}\right) = \sum_{i=1}^{N} \ln Nu \ln\left(\frac{Re}{\sqrt{y}}\right)$$

$$\sum_{i=1}^{N} c_1 \ln Gz \ln Pr + \sum_{i=1}^{N} c_2 \ln\left(\frac{Re}{\sqrt{y}}\right) \ln Pr + \sum_{i=1}^{N} c_3 \ln Pr \ln Pr + \sum_{i=1}^{N} c_4 \ln Gr \ln Pr + \sum_{i=1}^{N} c_5 \ln t_{hl} \ln Pr + \sum_{i=1}^{N} c_6 \ln sin\theta \ln Pr + \sum_{i=1}^{N} c_7 \ln sin\alpha \ln Pr + \sum_{i=1}^{N} c_8 \ln h_c \ln Pr = \sum_{i=1}^{N} \ln Nu \ln Pr$$

$$\sum_{i=1}^{N} c_1 \ln Gz \ln Gr + \sum_{i=1}^{N} c_2 \ln\left(\frac{Re}{\sqrt{y}}\right) \ln Gr + \sum_{i=1}^{N} c_3 \ln Pr \ln Gr + \sum_{i=1}^{N} c_4 (\ln Gr)^2 + \sum_{i=1}^{N} c_5 \ln t_{hl} \ln Gr + \sum_{i=1}^{N} c_6 \ln sin\theta \ln Gr + \sum_{i=1}^{N} c_7 \ln sin\alpha \ln Gr + \sum_{i=1}^{N} c_8 \ln h_c \ln Gr = \sum_{i=1}^{N} \ln Nu \ln Gr$$

$$\sum_{i=1}^{N} c_1 \ln Gz \ln t_{hl} + \sum_{i=1}^{N} c_2 \ln\left(\frac{Re}{\sqrt{y}}\right) \ln t_{hl} + \sum_{i=1}^{N} c_3 \ln Pr \ln t_{hl} \sum_{i=1}^{N} c_4 \ln Gr \ln t_{hl} + \sum_{i=1}^{N} c_5 \ln t_{hl} \ln t_{hl} + \sum_{i=1}^{N} c_6 \ln sin\theta \ln t_{hl} + \sum_{i=1}^{N} c_7 \ln sin\alpha \ln t_{hl} + \sum_{i=1}^{N} c_8 \ln h_c \ln t_{hl} = \sum_{i=1}^{N} \ln Nu \ln t_{hl}$$

$$\sum_{i=1}^{N} \ln Gz \ln sin\theta \quad + \sum_{i=1}^{N} c_2 \ln\left(\frac{Re}{\sqrt{y}}\right) \ln sin\theta + \sum_{i=1}^{N} c_3 \ln Pr \ln sin\theta + \sum_{i=1}^{N} c_4 \ln Gr \ln sin\theta + \sum_{i=1}^{N} c_5 \ln t_{hl} \ln sin\theta + \sum_{i=1}^{N} c_6 \ln sin\theta \ln sin\theta + \sum_{i=1}^{N} c_7 \ln sin\alpha \ln sin\theta + \sum_{i=1}^{N} c_8 \ln h_c \ln sin\theta = \sum_{i=1}^{N} \ln Nu \ln sin\theta$$

$$\sum_{i=1}^{N} c_1 \ln Gz \ln sin\alpha + \sum_{i=1}^{N} c_2 \ln\left(\frac{Re}{\sqrt{y}}\right) \ln sin\alpha + \sum_{i=1}^{N} c_3 \ln Pr \ln sin\alpha + \sum_{i=1}^{N} c_4 \ln Gr \ln sin\alpha + \sum_{i=1}^{N} c_5 \ln t_{hl} \ln sin\alpha + \sum_{i=1}^{N} c_6 \ln sin\theta \ln sin\alpha + \sum_{i=1}^{N} c_7 \ln sin\alpha \ln sin\alpha + \sum_{i=1}^{N} c_8 \ln h_c \ln sin\alpha = \sum_{i=1}^{N} \ln Nu \ln sin\alpha$$

$$\sum_{i=1}^{N} c_1 \ln Gz \ln h_c + \sum_{i=1}^{N} c_2 \ln\left(\frac{Re}{\sqrt{y}}\right) \ln h_c + \sum_{i=1}^{N} c_3 \ln Pr \ln h_c + \sum_{i=1}^{N} c_4 \ln Gr \ln h_c + \sum_{i=1}^{N} c_5 \ln t_{hl} \ln h_c + \sum_{i=1}^{N} c_6 \ln sin\theta \ln h_c + \sum_{i=1}^{N} c_7 \ln sin\alpha \ln h_c + \sum_{i=1}^{N} c_8 \ln h_c \ln h_c = \sum_{i=1}^{N} \ln Nu \ln h_c$$

$$\Delta = \begin{vmatrix}
\sum_{i=1}^{N}(\ln Gz)^2 & \sum_{i=1}^{N}\ln\left(\frac{Re}{\sqrt{y}}\right)\ln Gz & \sum_{i=1}^{N}\ln Pr\ln Gz & \sum_{i=1}^{N}\ln Gr\ln Gz & \sum_{i=1}^{N}\ln t_{hl}\ln Gz & \sum_{i=1}^{N}\ln sin\theta\ln Gz & \sum_{i=1}^{N}\ln sin\alpha\ln Gz & \sum_{i=1}^{N}\ln h_c\ln Gz \\
\sum_{i=1}^{N}\ln Gz\ln\left(\frac{Re}{\sqrt{y}}\right) & \sum_{i=1}^{N}\left(\ln\left(\frac{Re}{\sqrt{y}}\right)\right)^2 & \sum_{i=1}^{N}\ln Pr\ln\left(\frac{Re}{\sqrt{y}}\right) & \sum_{i=1}^{N}\ln Gr\ln\left(\frac{Re}{\sqrt{y}}\right) & \sum_{i=1}^{N}\ln t_{hl}\ln\left(\frac{Re}{\sqrt{y}}\right) & \sum_{i=1}^{N}\ln sin\theta\ln\left(\frac{Re}{\sqrt{y}}\right) & \sum_{i=1}^{N}\ln sin\alpha\ln\left(\frac{Re}{\sqrt{y}}\right) & \sum_{i=1}^{N}\ln h_c\ln\left(\frac{Re}{\sqrt{y}}\right) \\
\sum_{i=1}^{N}\ln Gz\ln Pr & \sum_{i=1}^{N}\ln\left(\frac{Re}{\sqrt{y}}\right)\ln Pr & \sum_{i=1}^{N}(\ln Pr)^2 & \sum_{i=1}^{N}\ln Gr\ln Pr & \sum_{i=1}^{N}\ln t_{hl}\ln Pr & \sum_{i=1}^{N}\ln sin\theta\ln Pr & \sum_{i=1}^{N}\ln sin\alpha\ln Pr & \sum_{i=1}^{N}\ln h_c\ln Pr \\
\sum_{i=1}^{N}\ln Gz\ln Gr & \sum_{i=1}^{N}\ln\left(\frac{Re}{\sqrt{y}}\right)\ln Gr & \sum_{i=1}^{N}\ln Pr\ln Gr & \sum_{i=1}^{N}(\ln Gr)^2 & \sum_{i=1}^{N}\ln t_{hl}\ln Gr & \sum_{i=1}^{N}\ln sin\theta\ln Gr & \sum_{i=1}^{N}\ln sin\alpha\ln Gr & \sum_{i=1}^{N}\ln h_c\ln Gr \\
\sum_{i=1}^{N}\ln Gz\ln t_{hl} & \sum_{i=1}^{N}\ln\left(\frac{Re}{\sqrt{y}}\right)\ln t_{hl} & \sum_{i=1}^{N}\ln Pr\ln t_{hl} & \sum_{i=1}^{N}\ln Gr\ln t_{hl} & \sum_{i=1}^{N}(\ln t_{hl})^2 & \sum_{i=1}^{N}\ln sin\theta\ln t_{hl} & \sum_{i=1}^{N}\ln sin\alpha\ln t_{hl} & \sum_{i=1}^{N}\ln h_c\ln t_{hl} \\
\sum_{i=1}^{N}\ln Gz\ln sin\theta & \sum_{i=1}^{N}\ln\left(\frac{Re}{\sqrt{y}}\right)\ln sin\theta & \sum_{i=1}^{N}\ln Pr\ln sin\theta & \sum_{i=1}^{N}\ln Gr\ln sin\theta & \sum_{i=1}^{N}\ln t_{hl}\ln sin\theta & \sum_{i=1}^{N}(\ln sin\theta)^2 & \sum_{i=1}^{N}\ln sin\alpha\ln sin\theta & \sum_{i=1}^{N}\ln h_c\ln sin\theta \\
\sum_{i=1}^{N}\ln Gz\ln sin\alpha & \sum_{i=1}^{N}\ln\left(\frac{Re}{\sqrt{y}}\right)\ln sin\alpha & \sum_{i=1}^{N}\ln Pr\ln sin\alpha & \sum_{i=1}^{N}\ln Gr\ln sin\alpha & \sum_{i=1}^{N}\ln t_{hl}\ln sin\alpha & \sum_{i=1}^{N}\ln sin\theta\ln sin\alpha & \sum_{i=1}^{N}(\ln sin\alpha)^2 & \sum_{i=1}^{N}\ln h_c\ln sin\alpha \\
\sum_{i=1}^{N}\ln Gz\ln h_c & \sum_{i=1}^{N}\ln\left(\frac{Re}{\sqrt{y}}\right)\ln h_c & \sum_{i=1}^{N}\ln Pr\ln h_c & \sum_{i=1}^{N}\ln Gr\ln h_c & \sum_{i=1}^{N}\ln t_{hl}\ln h_c & \sum_{i=1}^{N}\ln sin\theta\ln h_c & \sum_{i=1}^{N}\ln sin\alpha\ln h_c & \sum_{i=1}^{N}(\ln h_c)^2
\end{vmatrix}$$

$$B = \left(\sum_{i=1}^{N}\ln Nu\ln Gz \sum_{i=1}^{N}\ln Nu\ln\left(\frac{Re}{\sqrt{y}}\right)\sum_{i=1}^{N}\ln Nu\ln Pr \sum_{i=1}^{N}\ln Nu\ln Gr \sum_{i=1}^{N}\ln Nu\ln t_{hl} \sum_{i=1}^{N}\ln Nu\ln sin\theta \sum_{i=1}^{N}\ln Nu\ln sin\alpha \sum_{i=1}^{N}\ln Nu\ln h_c\right)^{-1}$$

For individual integral spiral corrugation and twisted-tape with oblique teeth, friction factors are given by:

$$f\,\mathrm{Re} = 3.309228\,\mathrm{Re}^{0.455395}(\sin\alpha)^{0.597974}h_c^{-0.51379}$$

$$f\,\mathrm{Re} = 0.419118\left(\frac{\mathrm{Re}}{\sqrt{y}}\right)^{0.627715}(\sin\vartheta)^{-1.40515}t_{hl}^{-0.60056}$$

The corresponding Nusselt number correlations are givenby:

$$Nu_m = 5.172Gz^{0.1887}\,\mathrm{Re}^{0.4228}\,\mathrm{Pr}^{-0.05846}Gr^{-0.19504}h_c^{0.03819}(\sin\alpha)^{-0.03295}\times\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

$$Nu_m = 5.172Gz^{-0.2683}\left(\frac{\mathrm{Re}}{\sqrt{y}}\right)^{0.8115}Gr^{-0.3689}\,\mathrm{Pr}^{0.3901}t_{hl}^{0.001842}(\sin\vartheta)^{-0.3629}\times\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

All correlations have been developed by log-linear regression analysis. The correlations predict most of the experimental data within ± 20 per cent.

IV. Conclusions

The experimental friction factor and Nusselt number data for laminar flow through a circular duct having only spiral corrugations and only twisted tape with oblique teeth have been considered. Predictive friction factor and Nusselt number correlations have also been developed and presented. These correlations predict data satisfactorily. This research finding is useful in manufacturing better heat exchangers used in chemical industries

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