Abstract—This study considers a dynamic lot-sizing problem for remanufacturing systems that consist of a single disassembly, parallel reprocessing and a single reassembly workstations. The problem is to determine the disassembly, reprocessing and reassembly lot-sizes that satisfy the remanufactured product demands while satisfying the capacity of each facility over a planning horizon for the objective of minimizing the sum of setup, operation and inventory holding costs. A mixed integer programming model is developed to represent the problem mathematically. Then, three heuristics are proposed that fixes a portion of binary variables and solves the resulting problems iteratively. Computational experiments were done on various test instances and the test results are reported.

Index Terms—remanufacturing, lot-sizing, fix-and-optimize.

I. INTRODUCTION

Remanufacturing is defined as recycling by manufacturing new products from end-of-use/life products, i.e. reprocessing end-of-use/life products in such a way that their qualities are as good as new in the aspects of appearance, reliability and performance. As explained in Steinhilper [1], remanufacturing is different from repair in that it is an industrialized process, not a simple mechanical work, with an overall restoration to likely new conditions after products are disassembled completely.

A typical remanufacturing system consists of three dependent processes, i.e. disassembly, reprocessing and reassembly. An end-of-use/life product is separated into its components in the disassembly process, and then, the usable ones are cleaned, reconditioned and tested in the reprocessing process. Finally, in the reassembly process, reprocessed and new components, if required, are assembled into the remanufactured product fully equivalent in performance and ready for use by customers.

As various decision problems in remanufacturing systems, we focus on the planning problem that determines the lot-sizes that satisfy the demands of remanufactured products, i.e. how much and when to disassemble end-of-life products, reprocess their components and reassemble remanufactured products. See Guide [2] for the importance of planning in remanufacturing systems.

The previous studies on lot-sizing in remanufacturing systems can be classified into single-stage and multi-stage ones. Richter and Sombrutzki [3] define a dynamic lot-sizing problem for single-stage remanufacturing systems with return flows and suggest the optimal algorithms that modify the well-known Wagner/Whitin algorithm. Later, it is extended by Richter and Weber [4], Piñeyro and Viera [5] and Cunha and Melo [6]. The single-stage models have a critical limitation in that the detailed material flows through the three processes are not considered. To overcome the limitation, others suggest multi-stage models. Clegg et al. [7] suggest a linear programming model for the problem that determines disassembly and reassembly lot-sizes for a manufacturing system with remanufacturing capability for the objective of maximizing the total profit. Jayaraman [8] suggest another linear programming model for the problem that determines the amounts of disassembling, remanufacturing, disposing, acquiring end-of-use/life products with probabilistic quality levels over a planning horizon. Doh and Lee [9] propose linear programming relaxation heuristics for the problem of determining the number of end-of-use/life products to be disassembled and disposed of, the number of components to be reprocessed, disposed of and newly purchased, and the number of remanufactured products to be reassembled over a planning horizon for the objective of maximizing the total profit. Also, Ahn et al. [10] develop a dynamic lot-sizing algorithm that minimizes the sum of setup and inventory holding costs for a remanufacturing system with a single disassembly facility, parallel reprocessing facilities and a single reassembly facility.

In this study, we consider a multi-stage dynamic lot-sizing problem for a remanufacturing system with a single disassembly facility, parallel reprocessing facilities and a single reassembly facility. The problem is to determine disassembly, reprocessing and reassembly lot-sizes for the objective of minimizing the sum of setup, operation and inventory holding costs over a planning horizon. The problem is an extension of Ahn et al. [10] in that the capacity of each facility is explicitly considered. As in other lot-sizing problems, the capacity constraint must be considered, so that the resulting solutions are more realistic and applicable to real remanufacturing systems. A mixed integer programming model is developed to represent the problem mathematically. Then, due to the complexity of the problem, we propose the fix-and-optimize based heuristic algorithms that fix a portion of binary variables with a certain method and solve the resulting problems iteratively. Computational experiments were done on various test instances, and the results are reported.

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II. SYSTEM AND PROBLEM DESCRIPTIONS

A. System Description

As explained earlier, the remanufacturing system considered in this study consists of a single disassembly facility, parallel reprocessing facilities and a single reassembly facility. Fig. 1 shows the system configuration and its material flow.

As can be seen in the figure, end-of-use/life products are separated into their components at the single disassembly facility, and then each component is reprocessed at one of the dedicated parallel reprocessing facilities, i.e. each component can be reprocessed at its pre-specified reprocessing facility. Finally, the reprocessed components are reassembled into remanufactured products at the single reassembly facility.

B. Problem description

The problem considered in this study can be briefly described as follows. For the remanufacturing system with a single disassembly facility, parallel reprocessing facilities and a single reassembly facility, the problem is to determine disassembly, reprocessing and reassembly lot-sizes that satisfy the processing time capacity of each facility and the dynamic remanufactured product demands over a given planning horizon with discrete time periods for the objective of minimizing the sum of setup, operation and inventory holding costs.

In the problem, the lot-sizes imply the quantity of the product to be disassembled at the disassembly facility, the quantity of the components to be reprocessed at each reprocessing facility and the quantity of remanufactured product to be reassembled at the reassembly facility in each period of the planning horizon. The objective is to minimize the sum of setup, operation and inventory holding costs over the planning horizon. The setup costs are those required for preparing disassembly, reprocessing and reassembly operations. As in other lot-sizing models, the setup costs occur in a period if any operation is performed in that period. Also, the operation costs are required to perform the disassembly, reprocessing or reassembly operations. Finally, the inventory holding costs, computed by the end-of-period inventory levels, are incurred to store disassembled components, reprocessed components and remanufactured products over the planning horizon. It is assumed that the operation and inventory holding costs are linear in the amounts of operations and inventory levels, respectively. The problem has two constraints. First, the demand constraint is that the dynamic demands of remanufactured product must be satisfied over the planning horizon. It is assumed that the demands are deterministic and given in advance and shortages are not allowed. Also, the capacity constraints imply that there are upper limits on the available times to perform setup and operation at each of disassembly, reprocessing and reassembly facilities.

In this study, we consider the deterministic version of the problem, i.e. all parameters are deterministic and given in advance. The other assumptions made for the problem are: (a) end-of-use/life products can be supplied whenever ordered; (b) disassembly, reprocessing and reassembly operations can be carried out within a single time period; and (c) there are no defectives.

To represent the problem mathematically, a mixed integer programming model developed. Before describing the model, the notations used are summarized below.

Indices

\[ i \text{ components (reprocessing facilities), } i = 1, 2 \ldots I \]
\[ t \text{ periods, } t = 1, 2 \ldots T \]

Parameters

\[ s_{d}^i \text{ setup cost to disassemble end-of-use/life products in period } t \text{ ($)} \]
\[ s_{p}^i \text{ setup cost to reprocess component } i \text{ in period } t \text{ ($)} \]
\[ s_{r}^i \text{ setup cost to reassemble remanufactured products in period } t \text{ ($)} \]
\[ o_{d} \text{ operation cost to disassemble one unit of the end-of-use/life product in period } t \text{ ($/unit)} \]
\[ o_{p}^i \text{ operation cost to reprocess one unit of component } i \text{ in period } t \text{ ($/unit)} \]
\[ o_{r}^i \text{ operation cost to reassemble one unit of remanufactured product in period } t \text{ ($/unit)} \]
\[ h_{a}^i \text{ inventory holding cost of disassembled component } i \text{ in period } t \text{ ($/unit-period)} \]
\[ h_{p}^i \text{ inventory holding cost of reprocessed component } i \text{ in period } t \text{ ($/unit-period)} \]
\[ h_{r}^i \text{ inventory holding cost of remanufactured products in period } t \text{ ($/unit-period)} \]
\[ d_{t} \text{ demand of remanufactured product in period } t \]
\[ \pi_{i} \text{ units of component } i \text{ obtained from disassembling one unit of the end-of-use/life product} \]
\[ u_{d}^i \text{ setup time to disassemble end-of-use/life products in period } t \text{ (hours)} \]
\[ u_{p}^i \text{ setup time to reprocess component } i \text{ in period } t \text{ (hours)} \]
\[ u_{r}^i \text{ setup time to reassemble remanufactured products in period } t \text{ (hours)} \]
\[ p_{d}^i \text{ operation time to disassemble one unit of the end-of-use/life product in period } t \text{ (hours)} \]
\[ p_{p}^i \text{ operation time to reprocess one unit of component } i \text{ in period } t \text{ (hours)} \]
\[ p_{r}^i \text{ operation time to reassemble one unit of remanufactured product in period } t \text{ (hours)} \]
\[ C_{d}^i \text{ available capacity at disassembly facility in period } t \text{ (hours)} \]
\[ C_{p}^i \text{ available capacity at reprocessing facility } i \text{ in period } t \text{ (hours)} \]
\[ C_{r}^i \text{ available capacity at reassembly facility in period } t \text{ (hours)} \]
\[ I_{0}^i \text{ initial inventory level of disassembled component } i \]
\[ I_{0}^p \text{ initial inventory level of reprocessed component } i \]
\[ I_{0}^r \text{ initial inventory level of remanufactured product} \]
\[ M \text{ large number} \]
Decision variables

\[y_{it}^d = 1\] if setup is done to disassemble end-of-use/life product in period \(t\), and 0 otherwise

\[y_{it}^p = 1\] if setup is done to reprocess component \(i\) in period \(t\), and 0 otherwise

\[y_{it}^r = 1\] if setup is done to reassemble remanufactured product in period \(t\), and 0 otherwise

\[x_{it}^d\] disassembly lot-size in period \(t\)

\[x_{it}^p\] reprocessing lot-size of component \(i\) in period \(t\)

\[x_{it}^r\] reassembly lot-size in period \(t\)

\[I_{it}^d\] inventory level of disassembled component \(i\) at the end of period \(t\)

\[I_{it}^p\] inventory level of reprocessed component \(i\) at the end of period \(t\)

\[I_{it}^r\] inventory level of remanufactured product at the end of period \(t\)

The mixed integer programming model is given below.

Minimize

\[
\sum_{i=1}^{T} \left( x_{it}^d y_{it}^d + x_{it}^p y_{it}^p + x_{it}^r y_{it}^r + \sum_{i=1}^{T} I_{it}^d I_{it}^d \right)
\]

subject to

\[I_{it}^d = I_{i,t-1}^d + \pi_i x_{it}^d - x_{it}^p \quad \text{for all } i = 1, 2, \ldots, I \text{ and } t = 1, 2, \ldots, T \quad (1)\]

\[I_{it}^p = I_{i,t-1}^p - \pi_i x_{it}^p + x_{it}^p + h_{it}^p - I_{it}^d \quad \text{for all } i = 1, 2, \ldots, I \text{ and } t = 1, 2, \ldots, T \quad (2)\]

\[I_{i}^r = I_{i,1}^r + x_{i,1}^r - x_{i,t}^r \quad \text{for all } i = 1, 2, \ldots, T \quad (3)\]

\[x_{it}^d \leq M \cdot y_{it}^d \quad \text{for all } t = 1, 2, \ldots, T \quad (4)\]

\[x_{it}^p \leq M \cdot y_{it}^p \quad \text{for all } i = 1, 2, \ldots, I \text{ and } t = 1, 2, \ldots, T \quad (5)\]

\[x_{it}^r \leq M \cdot y_{it}^r \quad \text{for all } t = 1, 2, \ldots, T \quad (6)\]

\[u_{it}^d y_{it}^d + p_{it}^d x_{it}^d \leq C_{it}^d \quad \text{for all } i = 1, 2, \ldots, T \quad (7)\]

\[u_{it}^p y_{it}^p + p_{it}^p x_{it}^p \leq C_{it}^p \quad \text{for all } i = 1, 2, \ldots, I \text{ and } t = 1, 2, \ldots, T \quad (8)\]

\[u_{it}^r y_{it}^r + p_{it}^r x_{it}^r \leq C_{it}^r \quad \text{for all } t = 1, 2, \ldots, T \quad (9)\]

\[y_{it}^d, y_{it}^p, y_{it}^r \in \{0,1\} \quad \text{for all } i = 1, 2, \ldots, I \text{ and } t = 1, 2, \ldots, T \quad (10)\]

\[x_{it}^d, x_{it}^p, x_{it}^r \geq 0 \quad \text{for all } i = 1, 2, \ldots, I \text{ and } t = 1, 2, \ldots, T \quad (11)\]

\[I_{it}^d, I_{it}^p, I_{it}^r \geq 0 \quad \text{for all } i = 1, 2, \ldots, I \text{ and } t = 1, 2, \ldots, T \quad (12)\]

The objective represents the sum of setup, operation and inventory holding costs over the planning horizon. Constraints (1), (2) and (3) represent the inventory balances of disassembled components, reprocessed components and remanufactured products, respectively. For example, constraint (1) implies that the inventory level of a disassembled component at the end of a period is the inventory in a period before, increased by the amount of the component obtained from disassembling the products in that period and decreased by the amount reprocessed in that period. Also, the demand requirements are represented by constraint (3). Constraints (4), (5) and (6) ensure that a setup in a period occurs whenever at least one disassembly, reprocessing or reassembly operation is performed in that period. Constraints (7), (8) and (9) represent the capacity constraints. Finally, constraints (10), (11) and (12) represent the conditions of decision variables.

The problem can be solved using a commercial software package. However, this requires an excessive amount of time. In fact, we can easily see that the problem is NP-hard since the formulation has the knapsack constraints and binary decision variables.

III. Solution Approach

To solve the problem, this study adopts the fix-and-optimize approach since it gives good solutions for various lot-sizing problems. See Helber and Sahling [11] and Helber et al. [12] for successful applications of the fix-and-optimize approach to other lot-sizing problems.

The fix-and-optimize solution approach proposed in this study works as follows. First, an initial solution is obtained by solving the model [P] after fixing all the binary setup variables to 1, i.e. \(y_{it}^d, y_{it}^p\) and \(y_{it}^r = 1\) for all \(i, t\). Second, some of the setup variables are selected using a variable selection method and then, they are freed to binary variables. For this purpose, we propose three methods to select the setup variables to be freed because the performance of the fix-and-optimize heuristic highly depends on the method to select the binary variables to be freed. Third, the selected setup variables are freed to binary ones and an incumbent solution is obtained by solving the resulting problem with fixed and freed variables optimally. Here, the unnecessary positive setup variables are set to 0 to reduce unnecessary setup costs, i.e. set \(y_{it}^d, y_{it}^p, y_{it}^r = 0\) if \(x_{it}^d, x_{it}^p, x_{it}^r = 0\), and the solution is updated if the incumbent solution improves the current one. Finally, the algorithm is stopped if there is no improvement for certain consecutive iterations.

This study proposes three variable selection methods based on the characteristics of the remanufacturing system considered in this study. The variable selection methods are explained below. Recall that \(I \) and \( T \) denote the numbers of reprocessing facilities and periods, respectively.

Partial-period method: From disassembly to reassembly facility, \([T/2]\) binary setup variables are selected for each facility in the non-increasing order of setup costs over the planning horizon, where \([\cdot]\) is the smallest integer larger than or equal to \(\cdot\). Then, in the next iteration, \(T - [T/2]\) setup variables are selected and hence variable selections are done two times for each facility. For example if there are 8 reprocessing facilities and 8 periods, i.e. \(I = 8\) and \(T = 8\), 4 binary setup variables are freed at each iteration, i.e. \([T/2] = [8/2] = 4\), which results in 20 iterations in maximum.

Full-period method: From disassembly to reassembly facility, all the binary setup variables are selected for each facility over the planning horizon, and hence variable selection is done one time for each facility. In the same example with 8 reprocessing facilities and 8 periods, there exist 10 iterations in maximum.

Overlapped-period method: From facility combination 1 to \(I\), \(u\) consecutive setup variables are selected from the first period of the planning horizon while overlapping the last \(u\) variables in
the next iteration. Here, facility combination \( i \) is defined as the set of disassembly facility, reprocessing facility \( i \), reassembly facility.

The detailed procedure of the fix-and-optimize solution approach is given below. Recall that three fix-and-optimize heuristics are proposed according to the three methods to select the setup variables to be freed.

**Procedure 1.** (Fix-and-optimize solution approach)

**Step 1.** (Initialization) Obtain an initial solution by solving the problem \([P]\) after fixing all the binary setup variables to 1, i.e., \( y_{i,t}^b \) and \( y_{i}^r \) for all \( i \) and \( t \).

**Step 2.** (Selecting free setup variables) Select the binary setup variables to be freed using one of the variable selection methods explained earlier.

**Step 3.** (Obtaining an incumbent solution) Obtain an incumbent solution by solving the problem obtained after freeing the selected setup variables optimally and set the unnecessary positive setup variables to 0. Maintain the best setup patterns at the next iteration.

**Step 4.** (Updating the solution and terminating the algorithm) Update the solution if the incumbent solution improves the current one. Stop the algorithm if there is no improvement for a certain consecutive number of iterations. Otherwise, go to Step 2.

**IV. COMPUTATIONAL RESULTS**

To test the three variants of the fix-and-optimize heuristic, computational experiments were done on various test instances and the results are reported in this section. The heuristics were coded in C++ with CPLEX 12.5 solver and the tests were done on a personal computer with an Intel Core i5 processor operating at 2.8 GHz clock speed and 4 GB of main memory.

In this test, we use two performance measures: (a) percentage deviations from optimal solution values or lower bounds; and (b) CPU seconds. Here, lower bounds were used when CPLEX could not give the optimal solutions within 3600 seconds. More formally, the percentage deviation from optimal solution value or lower bound for an instance is calculated as

\[
\frac{(C_h - C_b)}{C_h} \times 100
\]

where \( C_h \) is the solution value obtained from heuristic \( h \) and \( C_b \) is the optimal solution value or lower bounds obtained by solving the integer programming model using the CPLEX.

For the test, 10 instances were generated randomly for some combinations of 7 levels of the number of components (5, 10, 15, 20, 30, 40 and 50), 6 levels of the number of periods (5, 10, 15, 20, 30 and 40) and 3 levels of capacity tightness (loose, regular and tight). Here, the test instances were classified into the small-sized ones when the optimal solutions could be obtained using CPLEX within 3600 seconds and the large-sized ones when the optimal solutions could not be obtained. The detailed data were generated based on those of Jayaraman [8]. Also, the algorithms were terminated when there is no improvement for 10 (20, 30, 40 and 50) consecutive iterations for the instances with 5 and 10 (15/20, 30, 40 and 50) components. Also, in the overlapped-period method, parameters \( u \) and \( v \) were set to \([7/2]\) and 2 using the results of preliminary experiments.

Test results for the small-sized instances are summarized in Table I that shows the aggregated percentage deviations from the optimal solution values. It can be seen from the table that the fix-and-optimize heuristics give near optimal solutions. Among the three variable selection methods, the overlapped-period method outperforms the others since it explores the larger solution spaces in a broader way. In fact, the overall average percentage gaps of the best fix-and-optimize heuristic were 1.69%, 1.54% and 1.13% for the test instances with regular, loose and tight capacities, respectively. Also, the best heuristic gave smaller range between the maximum and the minimum percentage gaps than the others, which shows the stability of the overlapped-period based heuristic algorithm. Also, interestingly, the performances of the fix-and-optimize heuristics get better as the capacity tightness increases.

<table>
<thead>
<tr>
<th>Variable selection methods</th>
<th>Partial-period</th>
<th>Full-period</th>
<th>Overlapped-period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular capacities (0.26, 6.07) *</td>
<td>(0.25, 5.37)</td>
<td>(0.10, 4.24)</td>
<td></td>
</tr>
<tr>
<td>Loose capacities (0.62, 8.00)</td>
<td>(0.59, 8.00)</td>
<td>(0.10, 4.64)</td>
<td></td>
</tr>
<tr>
<td>Tight capacities (0.22, 6.50)</td>
<td>(0.18, 6.32)</td>
<td>(0.06, 2.83)</td>
<td></td>
</tr>
</tbody>
</table>

Average percentage deviation from the optimal solution values out of all test instances (minimum and maximum deviations in parenthesis)

Table II shows the test results for the large-sized instances. As can be seen in the table, the test results are similar to those for small-sized ones. In other words, the overlapped-period based fix-and-optimize heuristic gave better solutions than the others, and its overall average percentage gaps from the lower bounds were 3.39%, 3.64% and 3.03% for the test instances with regular, loose and tight capacities, respectively. Also, it gave smaller range between the maximum and the minimum percentage gaps than the others.

<table>
<thead>
<tr>
<th>Variable selection methods</th>
<th>Partial-period</th>
<th>Full-period</th>
<th>Overlapped-period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular capacities (3.55, 8.00)</td>
<td>(3.27, 7.61)</td>
<td>(2.21, 4.65)</td>
<td></td>
</tr>
<tr>
<td>Loose capacities (4.68, 9.36)</td>
<td>(4.61, 9.23)</td>
<td>(2.15, 4.81)</td>
<td></td>
</tr>
<tr>
<td>Tight capacities (2.60, 6.70)</td>
<td>(2.90, 6.33)</td>
<td>(1.75, 4.82)</td>
<td></td>
</tr>
</tbody>
</table>

Finally, the overlapped-period based fix-and-optimize heuristic required longer CPU seconds than the others due to its larger search space. Nevertheless, it gave the solutions within 385 seconds for the largest test instances with 50 components and 40 periods. In summary, it can be concluded from the test results that the fix-and-optimize approach, especially with the overlapped-period variable selection method, is effective in terms of solution quality and computation time for capacitated dynamic lot-sizing for remanufacturing systems with a single
disassembly facility, parallel reprocessing facilities and a single reassembly facility.

V. CONCLUSION

We considered the capacitated dynamic lot-sizing problem for a remanufacturing system with a single disassembly facility, parallel reprocessing facilities and a single reassembly facility. The problem is to determine disassembly, reprocessing and reassembly lot-sizes for the objective of minimizing the sum of setup, operation and inventory holding costs while satisfying the remanufactured product demands over a planning horizon. In particular, the capacity of each facility was explicitly considered. A mixed integer programming model was developed to describe the problem mathematically, and then three variants of the fix-and-optimize solution approach were proposed in which the solutions are obtained by fixing a portion of binary setup variables and solving the resulting reduced problems in an iterative manner. Computational experiments were done on various test instances and the test results showed that the overlapped-period based heuristic gives near optimal solutions within a reasonable amount of computation time. This study can be extended in several ways. First, it is needed to develop an optimal algorithm after characterizing the optimal solution properties. Second, the model can be extended to the one with multiple product types. Third, the uncertainty, such as defective components, safety stocks and other parameters, is an important consideration in remanufacturing systems. Finally, case studies are needed to show the applicability of the model and solution algorithms.

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